A NEW FORM OF RECURSION

COMP 210 – 28 OCT 2005

SCHEME: THE STORY SO FAR

- Values
  - simple (numbers, symbols, empty)
  - compound (structures, lists)
  - functions (lambda)
- Language
  - Ways to work with values
  - define, cond, local
  - primitive functions

WHAT CAN WE DO?

- Simple math of the $f(x) = x^2$ variety

```
(define (f x) (* x x))
```

- Structural recursion

```
;; A list is empty or (cons X list)
```

```
(define (f a-list)            ;; f : [X] → ?
  (cond
    [(empty? a-list) ...]
    [else ... (first a-list) ... (f ... (rest a-list) ...)]))
```

SO CAN WE DO ANYTHING?

- Yes, we can compute anything!
  - (As long as it's simple math.)
    - (Or walking down a list.)
      - (Or a family tree.)
        - (Or counting natural numbers down to 'Zero.)
- Is there nothing else?

WHAT ABOUT

GENIUS

INSPIRATION

CLEVERNESS

(ETC.)

EXPAND YOUR MIND

- And consider new types of computation

  …which do not fit "simple math" *or* "structural recursion".

PROBLEM #1: PHYSICS

- Can we figure out where an airborne object will hit the ground?
- High school phyiscs: modeling motion

$$x_{n+1} = x_n + \Delta x$$

(read Δx as "velocity")

- But if we have acceleration, we need to model how it changes velocity:

$$\Delta x_{n+1} = \Delta x_n + \Delta(\Delta x)$$

(read $\Delta(\Delta x)$ as "acceleration")

DEMO #1: LOOSE CANNONS

(ka-boom!)

THE SIMULATION

```
(define-struct obj (x y vx vy))
; sim : obj -> true
; repeatedly apply velocity to position, accel to velocity
; simulation stops when o hits the "ground" (y=0)
; assume fixed gravity in the y direction of -10 m/s
(define (sim o)
  (cond
    [(< (obj-y o) 0) true]        ; stop when we hit the ground
    [else (and
            (draw o)
            (sim (make-obj
                   (+ (obj-x o) (obj-vx o))     ; x_{n+1} = x_n + Δx
                   (+ (obj-y o) (obj-vy o))     ; y_{n+1} = y_n + Δy
                   (obj-vx o)
                   (+ (obj-vy o) -10)))))]))    ; Δy_{n+1} = Δy_n + Δ(Δy)
```

WHERE WAS THE TEMPLATE?

- We didn't know how to write one
  - There's no data definition for "physics simulation"
  - ...so it can't be structural recursion
- But it IS recursive.
  - Evidence: sim called sim again.
  - So what do we call it?
- The book calls it

"GENERATIVE RECURSION"

TEMPLATE FOR THE CANNON SIMULATION

- Instead of a cond based on structure:
  - A cond based on an *idea* for simulating things hitting the "ground"
    1. Keep moving an object
    2. If the object's y-value goes below 0, stop

- We can write a template for all simulations following this idea

```
(define (f o)
  (cond
    [(< (obj-y o) 0) ...]
    [else ... (f ...) ... ]))
```

THE STUDY OF

ALGORITHMS

CLEVER IDEAS FOR SOLVING PROBLEMS

A FUNDAMENTAL TOPIC IN COMPUTER SCIENCE

SYSTEMS, NETWORKS, LANGUAGES, CRYPTO, ROBOTICS, &C.

(OUTSIDE OF COMPUTER SCIENCE, TOO!)

DIVIDE AND CONQUER

- A general class of algorithms
- If your problem is easy to solve, solve it and stop
  - Otherwise, break it into strictly easier problems
    - And recursively examine those problems
      - If those problems are easy to solve …
        - (I think you get the idea)
- Conveniently expressed as recursive functions
- Here's a D&C algorithm for …

PROBLEM #2: SORTING

- THE PARTY HAT ALGORITHM
  - For a line of people to be sorted by birthday,
    - Pick someone to put on a party hat and shout out her birthday.
    - Everyone whose birthday comes before hers: move to her right.
    - Everyone whose birthday comes after: move to her left.
    - Start the game over with the people on her left.
    - Also start over with the people on her right.
    - At any point, if the line of people is empty, for goodness' sake, stop!

DEMO #2: THE PARTY HAT ALGORITHM AT WORK

(hopefully Dan remembered the party hats)

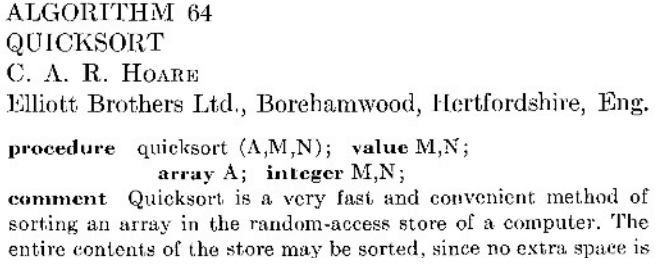
THAT'S A NEAT ALGORITHM

- This is actually an "old" algorithm (1960)

  (oh, and, it's not called "the party hat algorithm")

- It's called QUICKSORT*
  - ... because it's quick! It's a lot better than insertion sort, which we saw earlier.

* Invented by Sir C. A. R. Hoare, published in *Communications of the ACM*, July 1961



ALGORITHM 64
QUICKSORT
C. A. R. HOARE
Elliott Brothers Ltd., Borehamwood, Hertfordshire, Eng.

procedure quicksort (A,M,N); **value** M,N;
array A; **integer** M,N;
comment Quicksort is a very fast and convenient method of sorting an array in the random-access store of a computer. The entire contents of the store may be sorted, since no extra space is

TEMPLATE FOR ~~PARTY HAT~~ QUICKSORT

- Our algorithm said that we'd stop on an empty list, and perform a generative recursion otherwise

```
;; qsort-esque-func : [X] -> [X]
(define (qsort-esque-func L)
  (cond
    [(empty? L) ...]
    [else ... (qsort-esque-func ...) ... ]))
```

IMPLEMENTATION OF ~~PARTY HAT~~ QUICKSORT

```
;; qsort : [num] -> [num]
(define (qsort L)
  (local
     ((define (elements-before i L) (filter (lambda (x) (<= x i)) L))
      (define (elements-after i L)    (filter (lambda (x) (> x i)) L)))
    (cond
      [(empty? L) empty]
      [else
        (append
           (qsort (elements-before (first L) (rest L)))
           (list (first L))
           (qsort (elements-after (first L) (rest L))))])))
```

SOME FINAL THOUGHTS

- Is "generative recursion" really something fundamentally new?

- Algorithms are inventions, by people
  - They haven't all been found yet!

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